

# 1-2

# Points, Lines, and Planes

## Common Core State Standards

**G-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment . . .

**MP 1, MP 3, MP 4, MP 6**

**Objective** To understand basic terms and postulates of geometry



Does how the arrow goes through the board make sense?



### Getting Ready!

Make the figure at the right with a pencil and a piece of paper. Is the figure possible with a straight arrow and a solid board? Explain.



**MATHEMATICAL PRACTICES**

In this lesson, you will learn basic geometric facts to help you justify your answer to the Solve It.

**Essential Understanding** Geometry is a mathematical system built on accepted facts, basic terms, and definitions.

In geometry, some words such as *point*, *line*, and *plane* are undefined. Undefined terms are the basic ideas that you can use to build the definitions of all other figures in geometry. Although you cannot define undefined terms, it is important to have a general description of their meanings.



### Lesson Vocabulary

- point
- line
- plane
- collinear points
- coplanar
- space
- segment
- ray
- opposite rays
- postulate
- axiom
- intersection



### Key Concept Undefined Terms

#### Term Description

A **point** indicates a location and has no size.

A **line** is represented by a straight path that extends in two opposite directions without end and has no thickness. A line contains infinitely many points.

A **plane** is represented by a flat surface that extends without end and has no thickness. A plane contains infinitely many lines.

#### How to Name It

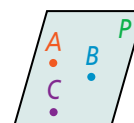
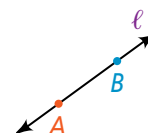
You can represent a point by a dot and name it by a capital letter, such as *A*.

You can name a line by any two points on the line, such as  $\overleftrightarrow{AB}$  (read “line *AB*”) or  $\overleftrightarrow{BA}$ , or by a single lowercase letter, such as line *ℓ*.

You can name a plane by a capital letter, such as plane *P*, or by at least three points in the plane that do not all lie on the same line, such as plane *ABC*.

#### Diagram

*A*



Points that lie on the same line are **collinear points**. Points and lines that lie in the same plane are **coplanar**. All the points of a line are coplanar.

## Think

**Why can figures have more than one name?**

Lines and planes are made up of many points. You can choose any two points on a line and any three or more noncollinear points in a plane for the name.



### Problem 1 Naming Points, Lines, and Planes

**A** What are two other ways to name  $\overleftrightarrow{QT}$ ?

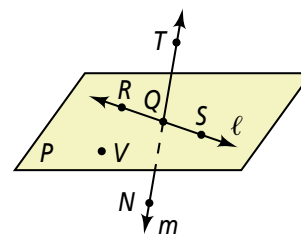
Two other ways to name  $\overleftrightarrow{QT}$  are  $\overleftrightarrow{TQ}$  and line  $m$ .

**B** What are two other ways to name plane  $P$ ?

Two other ways to name plane  $P$  are plane  $RQV$  and plane  $RSV$ .

**C** What are the names of three collinear points? What are the names of four coplanar points?

Points  $R$ ,  $Q$ , and  $S$  are collinear. Points  $R$ ,  $Q$ ,  $S$ , and  $V$  are coplanar.



**Got It? 1. a.** What are two other ways to name  $\overleftrightarrow{RS}$ ?

**b.** What are two more ways to name plane  $P$ ?

**c.** What are the names of three other collinear points?

**d.** What are two points that are *not* coplanar with points  $R$ ,  $S$ , and  $V$ ?

The terms *point*, *line*, and *plane* are not defined because their definitions would require terms that also need defining. You can, however, use undefined terms to define other terms. A geometric figure is a set of points. **Space** is the set of all points in three dimensions. Similarly, the definitions for *segment* and *ray* are based on points and lines.

## Take note

### Key Concept Defined Terms

#### Definition

A **segment** is part of a line that consists of two endpoints and all points between them.

A **ray** is part of a line that consists of one endpoint and all the points of the line on one side of the endpoint.

**Opposite rays** are two rays that share the same endpoint and form a line.

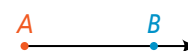
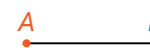
#### How to Name It

You can name a segment by its two endpoints, such as  $\overline{AB}$  (read “segment  $AB$ ”) or  $\overline{BA}$ .

You can name a ray by its endpoint and another point on the ray, such as  $\overrightarrow{AB}$  (read “ray  $AB$ ”). The order of points indicates the ray’s direction.

You can name opposite rays by their shared endpoint and any other point on each ray, such as  $\overrightarrow{CA}$  and  $\overrightarrow{CB}$ .

#### Diagram



## Plan

How do you make sure you name all the rays?

Each point on the line is an endpoint for a ray. At each point, follow the line both left and right to see if you can find a second point to name the ray.



### Problem 2 Naming Segments and Rays

**A** What are the names of the segments in the figure at the right?

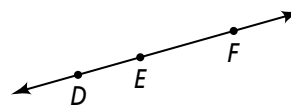
The three segments are  $\overline{DE}$  or  $\overline{ED}$ ,  $\overline{EF}$  or  $\overline{FE}$ , and  $\overline{DF}$  or  $\overline{FD}$ .

**B** What are the names of the rays in the figure?

The four rays are  $\overrightarrow{DE}$  or  $\overrightarrow{DF}$ ,  $\overrightarrow{ED}$ ,  $\overrightarrow{EF}$ , and  $\overrightarrow{FD}$  or  $\overrightarrow{FE}$ .

**C** Which of the rays in part (B) are opposite rays?

The opposite rays are  $\overrightarrow{ED}$  and  $\overrightarrow{EF}$ .



**Got It? 2. Reasoning**  $\overrightarrow{EF}$  and  $\overrightarrow{FE}$  form a line. Are they opposite rays? Explain.

A **postulate** or **axiom** is an accepted statement of fact. Postulates, like undefined terms, are basic building blocks of the logical system in geometry. You will use logical reasoning to prove general concepts in this book.

You have used some of the following geometry postulates in algebra. For example, you used Postulate 1-1 when you graphed equations such as  $y = 2x + 8$ . You graphed two points and drew the line through the points.

Take note

#### Postulate 1-1

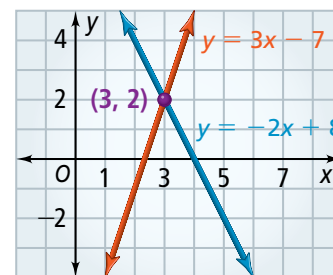
Through any two points there is exactly one line.

Line  $t$  passes through points  $A$  and  $B$ . Line  $t$  is the only line that passes through both points.



When you have two or more geometric figures, their **intersection** is the set of points the figures have in common.

In algebra, one way to solve a system of two equations is to graph them. The graphs of the two lines  $y = -2x + 8$  and  $y = 3x - 7$  intersect in a single point  $(3, 2)$ . So the solution is  $(3, 2)$ . This illustrates Postulate 1-2.

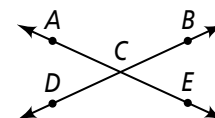


Take note

#### Postulate 1-2

If two distinct lines intersect, then they intersect in exactly one point.

$\overleftrightarrow{AE}$  and  $\overleftrightarrow{DB}$  intersect in point  $C$ .



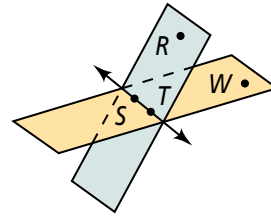
There is a similar postulate about the intersection of planes.

Take note

### Postulate 1-3

If two distinct planes intersect, then they intersect in exactly one line.

Plane  $RST$  and plane  $WST$  intersect in  $\overleftrightarrow{ST}$ .

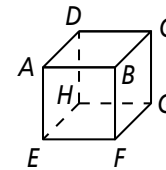


When you know two points that two planes have in common, Postulates 1-1 and 1-3 tell you that the line through those points is the intersection of the planes.



### Problem 3 Finding the Intersection of Two Planes

Each surface of the box at the right represents part of a plane. What is the intersection of plane  $ADC$  and plane  $BFG$ ?



#### Know

Plane  $ADC$  and plane  $BFG$

#### Need

The intersection of the two planes

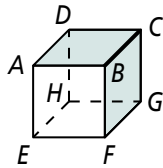
#### Plan

Find the points that the planes have in common.

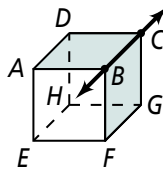
#### Think

Is the intersection a segment?

No. The intersection of the sides of the box is a segment, but planes continue without end. The intersection is a line.



Focus on plane  $ADC$  and plane  $BFG$  to see where they intersect.



You can see that both planes contain point  $B$  and point  $C$ .

The planes intersect in  $\overleftrightarrow{BC}$ .



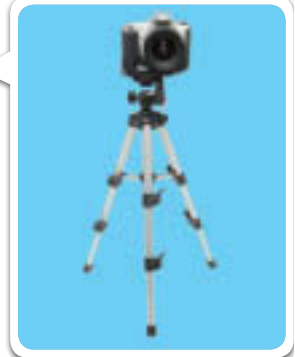
**Got It?** 3. a. What are the names of two planes that intersect in  $\overleftrightarrow{BF}$ ?

b. **Reasoning** Why do you only need to find two common points to name the intersection of two distinct planes?

When you name a plane from a figure like the box in Problem 3, list the corner points in consecutive order. For example, plane  $ADCB$  and plane  $ABCD$  are also names for the plane on the top of the box. Plane  $ACBD$  is not.

Photographers use three-legged tripods to make sure that a camera is steady. The feet of the tripod all touch the floor at the same time. You can think of the feet as points and the floor as a plane. As long as the feet do not all lie in one line, they will lie in exactly one plane.

This illustrates Postulate 1-4.

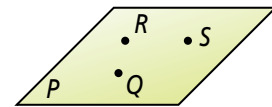


Take note

### Postulate 1-4

Through any three noncollinear points there is exactly one plane.

Points  $Q$ ,  $R$ , and  $S$  are noncollinear. Plane  $P$  is the only plane that contains them.



## Plan

How can you find the plane?

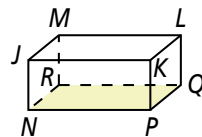
Try to draw all the lines that contain two of the three given points. You will begin to see a plane form.



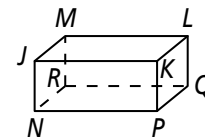
### Problem 4 Using Postulate 1-4

Use the figure at the right.

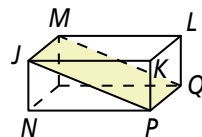
- A** What plane contains points  $N$ ,  $P$ , and  $Q$ ? Shade the plane.



The plane on the bottom of the figure contains points  $N$ ,  $P$ , and  $Q$ .



- B** What plane contains points  $J$ ,  $M$ , and  $Q$ ? Shade the plane.



The plane that passes at a slant through the figure contains points  $J$ ,  $M$ , and  $Q$ .



- Got It?** 4. a. What plane contains points  $L$ ,  $M$ , and  $N$ ? Copy the figure in Problem 4 and shade the plane.

- b. **Reasoning** What is the name of a line that is coplanar with  $\overleftrightarrow{JK}$  and  $\overleftrightarrow{KL}$ ?

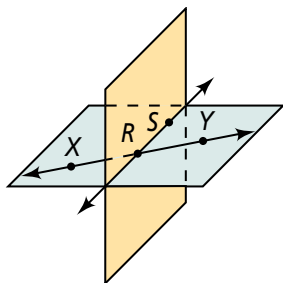


## Lesson Check

### Do you know HOW?

Use the figure at the right.

- What are two other names for  $\overleftrightarrow{XY}$ ?
- What are the opposite rays?
- What is the intersection of the two planes?



### Do you UNDERSTAND?



- Vocabulary** A segment has endpoints R and S. What are two names for the segment?
- Are  $\overrightarrow{AB}$  and  $\overrightarrow{BA}$  the same ray? Explain.
- Reasoning** Why do you use two arrowheads when drawing or naming a line such as  $\overleftrightarrow{EF}$ ?
- Compare and Contrast** How is naming a ray similar to naming a line? How is it different?



## Practice and Problem-Solving Exercises



### A Practice

Use the figure at the right for Exercises 8–11.

- What are two other ways to name  $\overleftrightarrow{EF}$ ?
- What are two other ways to name plane C?
- Name three collinear points.
- Name four coplanar points.

Use the figure at the right for Exercises 12–14.

- Name the segments in the figure.
- Name the rays in the figure.
- a. Name the pair of opposite rays with endpoint T.  
b. Name another pair of opposite rays.

Use the figure at the right for Exercises 15–26.

Name the intersection of each pair of planes.

- planes QRS and RSW
- planes UXV and WVS
- planes XWV and UVR
- planes TXW and TQU

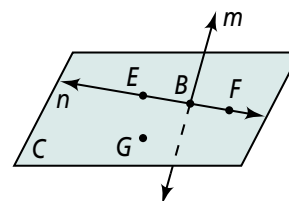
Name two planes that intersect in the given line.

- $\overleftrightarrow{QU}$
- $\overleftrightarrow{TS}$
- $\overleftrightarrow{XT}$
- $\overleftrightarrow{VW}$

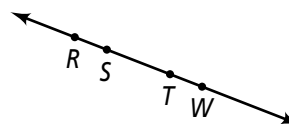
Copy the figure. Shade the plane that contains the given points.

- R, V, W
- U, V, W
- U, X, S
- T, U, V

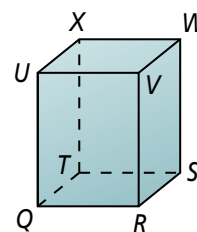
See Problem 1.



See Problem 2.



See Problem 3.



See Problem 4.

# B Apply

Postulate 1-4 states that any three noncollinear points lie in exactly one plane. Find the plane that contains the first three points listed. Then determine whether the fourth point is in that plane. Write *coplanar* or *noncoplanar* to describe the points.

27. Z, S, Y, C

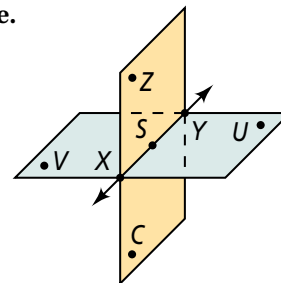
28. S, U, V, Y

29. X, Y, Z, U

30. X, S, V, U

31. X, Z, S, V

32. S, V, C, Y



If possible, draw a figure to fit each description. Otherwise, write *not possible*.

33. four points that are collinear

34. two points that are noncollinear

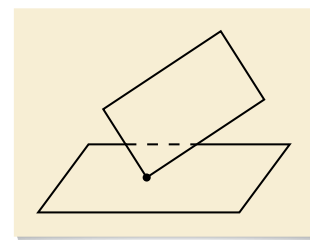
35. three points that are noncollinear

36. three points that are noncoplanar

37. **Open-Ended** Draw a figure with points B, C, D, E, F, and G that shows  $\overleftrightarrow{CD}$ ,  $\overleftrightarrow{BG}$ , and  $\overleftrightarrow{EF}$ , with one of the points on all three lines.

38. **Think About a Plan** Your friend drew the diagram at the right to prove to you that two planes can intersect in exactly one point. Describe your friend's error.

- How do you describe a plane?
- What does it mean for two planes to intersect each other?
- Can you define an endpoint of a plane?



39. **Reasoning** If one ray contains another ray, are they the same ray? Explain.

For Exercises 40–45, determine whether each statement is *always*, *sometimes*, or *never* true.

40.  $\overleftrightarrow{TQ}$  and  $\overleftrightarrow{QT}$  are the same line.

41.  $\overrightarrow{JK}$  and  $\overrightarrow{JL}$  are the same ray.

42. Intersecting lines are coplanar.

43. Four points are coplanar.

44. A plane containing two points of a line contains the entire line.

45. Two distinct lines intersect in more than one point.

46. Use the diagram at the right. How many planes contain each line and point?

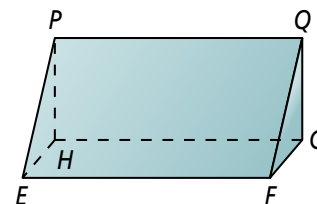
a.  $\overleftrightarrow{EF}$  and point G

b.  $\overleftrightarrow{PH}$  and point E

c.  $\overleftrightarrow{FG}$  and point P

d.  $\overleftrightarrow{EP}$  and point G

e. **Reasoning** What do you think is true of a line and a point not on the line? Explain. (*Hint*: Use two of the postulates you learned in this lesson.)



In Exercises 47–49, sketch a figure for the given information. Then state the postulate that your figure illustrates.

47.  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{EF}$  intersect in point  $C$ .
48. The noncollinear points  $A$ ,  $B$ , and  $C$  are all contained in plane  $N$ .
49. Planes  $LNP$  and  $MVK$  intersect in  $\overleftrightarrow{NM}$ .

- STEM** 50. **Telecommunications** A cell phone tower at point  $A$  receives a cell phone signal from the southeast. A cell phone tower at point  $B$  receives a signal from the same cell phone from due west. Trace the diagram at the right and find the location of the cell phone. Describe how Postulates 1-1 and 1-2 help you locate the phone.



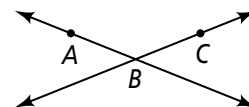
51. **Estimation** You can represent the hands on a clock at 6:00 as opposite rays. Estimate the other 11 times on a clock that you can represent as opposite rays.
52. **Open-Ended** What are some basic words in English that are difficult to define?

**Coordinate Geometry** Graph the points and state whether they are collinear.

53.  $(1, 1)$ ,  $(4, 4)$ ,  $(-3, -3)$       54.  $(2, 4)$ ,  $(4, 6)$ ,  $(0, 2)$       55.  $(0, 0)$ ,  $(-5, 1)$ ,  $(6, -2)$
56.  $(0, 0)$ ,  $(8, 10)$ ,  $(4, 6)$       57.  $(0, 0)$ ,  $(0, 3)$ ,  $(0, -10)$       58.  $(-2, -6)$ ,  $(1, -2)$ ,  $(4, 1)$

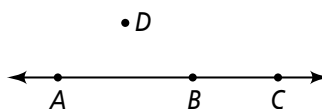


59. How many planes contain the same three collinear points? Explain.
60. How many planes contain a given line? Explain.
61. a. **Writing** Suppose two points are in plane  $P$ . Explain why the line containing the points is also in plane  $P$ .  
 b. **Reasoning** Suppose two lines intersect. How many planes do you think contain both lines? Use the diagram at the right and your answer to part (a) to explain your answer.



**Probability** Suppose you pick points at random from  $A$ ,  $B$ ,  $C$ , and  $D$  shown below. Find the probability that the number of points given meets the condition stated.

62. 2 points, collinear
63. 3 points, collinear
64. 3 points, coplanar



## Standardized Test Prep

SAT/ACT

65. Which geometric term is undefined?

(A) segment

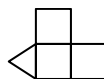
(B) collinear

(C) ray

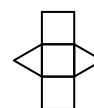
(D) plane

66. Which diagram is a net of the figure shown at the right?

(F)



(H)



(G)



(I)



67. You want to cut a block of cheese into four pieces. What is the least number of cuts you need to make?

(A) 2

(B) 3

(C) 4

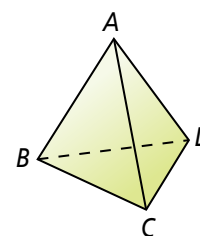
(D) 5

Short Response

68. The figure at the right is called a tetrahedron.

a. Name all the planes that form the surfaces of the tetrahedron.

b. Name all the lines that intersect at D.

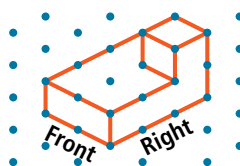


## Mixed Review

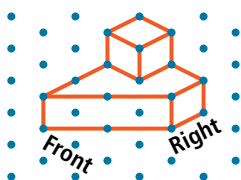
Make an orthographic drawing for each figure. Assume there are no hidden cubes.

See Lesson 1-1.

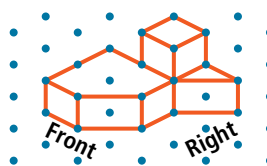
69.



70.



71.



Simplify each ratio.

72. 30 to 12

73.  $\frac{15x}{35x}$

74.  $\frac{n^2 + n}{4n}$

See p. 891.

**Get Ready!** To prepare for Lesson 1-3, do Exercises 75–80.

Simplify each absolute value expression.

See p. 892.

75.  $|-6|$

76.  $|3.5|$

77.  $|7 - 10|$

**Algebra** Solve each equation.

See p. 894.

78.  $x + 2x - 6 = 6$

79.  $3x + 9 + 5x = 81$

80.  $w - 2 = -4 + 7w$